

**Exercice 1**

a)  $\ln\left(\frac{1}{12}\right) = \ln 1 - \ln 12 = 0 - \ln(2^2 \times 3) = -\ln(2^2) - \ln 3 = \boxed{-2\ln 2 - \ln 3}$ .

b)  $\ln(3e^2) = \ln 3 + \ln(e^2) = \ln 3 + 2\ln e = \boxed{\ln 3 + 2}$ .

c)  $\ln(2\sqrt{3}) = \ln 2 + \ln(\sqrt{3}) = \boxed{\ln 2 + \frac{1}{2}\ln 3}$ .

d)  $\ln 648 = \ln(2^3 \times 3^4) = \ln(2^3) + \ln(3^4) = \boxed{3\ln 2 + 4\ln 3}$ .

**Exercice 2**

a)  $1 - \ln 2 = \ln e - \ln 2 = \boxed{\ln\left(\frac{e}{2}\right)}$ .

b)  $-2\ln 3 = -\ln(3^2) = -\ln 9 = \boxed{\ln\left(\frac{1}{9}\right)}$ .

c)  $2\ln 3 - 3\ln 2 = \ln(3^2) - \ln(2^3) = \ln 9 - \ln 8 = \boxed{\ln\left(\frac{9}{8}\right)}$ .

d)  $\frac{\ln 32}{5} = \frac{\ln(2^5)}{5} = \frac{5\ln 2}{5} = \boxed{\ln 2}$ .

**Exercice 3**

a)  $\ln(3 - 2x) = 5 \Leftrightarrow e^{\ln(3-2x)} = e^5 \Leftrightarrow 3 - 2x = e^5 \Leftrightarrow \boxed{x = \frac{3 - e^5}{2}}$ .

b)  $\ln(2x - 3) = \ln(x - 2) \Leftrightarrow 2x - 3 = x - 2$  et  $x - 2 > 0 \Leftrightarrow x = 1$  et  $x - 2 > 0$  : **aucune solution!**

c)  $\ln(x - 2) + \ln(x + 3) = \ln(5x - 9) \Leftrightarrow e^{\ln(x-2)+\ln(x+3)} = e^{\ln(5x-9)} \Leftrightarrow e^{\ln(x-2)} \times e^{\ln(x+3)} = e^{\ln(5x-9)} \Leftrightarrow (x - 2)(x + 3) = 5x - 9$  et  $x - 2 > 0$  et  $x + 3 > 0 \Leftrightarrow x^2 - 4x + 3 = 0$  et  $x - 2 > 0$  et  $x + 3 > 0 \Leftrightarrow x \in \{1; 3\}$  et  $x - 2 > 0$  et  $x + 3 > 0 \Leftrightarrow \boxed{x = 3}$ .

**Exercice 4**

a)  $e^{2\ln x} = (e^{\ln x})^2 = \boxed{x^2}$ .

b)  $10^{\ln x} = e^{\ln 10 \ln x} = \boxed{x^{\ln 10}}$ .

c)  $x^2 \sqrt[3]{x} = x^2 \times x^{\frac{1}{3}} = x^{2+\frac{1}{3}} = \boxed{x^{\frac{7}{3}}}$ .

d)  $\frac{\sqrt[3]{\sqrt[7]{x}}}{x} = \frac{(x^{\frac{1}{7}})^{\frac{1}{3}}}{x} = \frac{x^{\frac{1}{7} \times \frac{1}{3}}}{x} = \frac{x^{\frac{1}{21}}}{x^1} = x^{\frac{1}{21}-1} = \boxed{x^{-\frac{20}{21}}}$ .

**Exercice 5**

a)  $2^x = 10 \Leftrightarrow e^{x \ln 2} = 10 \Leftrightarrow x \ln 2 = \ln 10 \Leftrightarrow \boxed{x = \frac{\ln 10}{\ln 2}}$ .

b)  $\sqrt[3]{x} = \ln 2 \Leftrightarrow x^{\frac{1}{3}} = \ln 2 \Leftrightarrow x = (\ln 2)^3$  et  $x > 0 \Leftrightarrow \boxed{x = (\ln 2)^3}$ .

c)  $2^{x-1} + 2^{2-x} = 3 \Leftrightarrow \frac{2^x}{2} + \frac{4}{2^x} = 3 \Leftrightarrow \frac{X}{2} + \frac{4}{X} = 3$  avec  $X = 2^x \Leftrightarrow X^2 - 6X + 8 = 0$  avec  $X = 2^x \neq 0 \Leftrightarrow X \in \{2; 4\}$  avec  $X = 2^x \neq 0 \Leftrightarrow 2^x = 2$  ou  $2^x = 4 \Leftrightarrow \boxed{x = 1 \text{ ou } x = 2}$ .